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ACOUSTIC SURFACE WAVE ATTENUATION CALCULATIONS

William R. Jones William R. Smith Donald Perry

Hughes Aircraft Company Ground Systems Group P.O. Bex 3310 Fullerton, California 92634

Contract No. F19628-70-C-0027

Project No. 5635 Task No. 563503 Work Unit No. 56350301

Scientific Report No. 1

3 June 1970

Contract Monitor: Andrew J. Slobodnik, 1st Lt. USAF Microwave Physics Laboratory

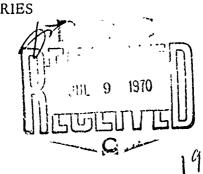
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OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE

BEDFORD, MASSACHUSETTS 01730

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ABSTRACT

Additions to and modifications of a computer program for the numerical study of piezoelectric surface wave propagation are described. The main addition to the program is the calculation of surface wave attenuation using an expression based on a perturbation analysis and the viscosity tensor which accounts for losses due to the Akheiser mechanism. Other additions and modifications include the evaluation of stored energy expressions and changes in the format and normalization of the output data.

This report describes additions to and modifications of a previously written computer program (1) for the numerical study of piezcelectric surface wave propagation. The extended program adds the calculation of a surface wave attenuation coefficient α_p , representing mechanical losses, for a general anisotropic, piezoelectric crystal with arbitrary surface cut and propagation direction (including the loss effects in the metal overlay where appropriate).

Mechanical loss can be accounted for by introducing complex elastic tensor coefficients c at the outset. The resultant complex surface wave velocity would thus specify the attenuation constant as well as the phase constant. This procedure is computationally disadvantageous because the loss (imaginary) terms in the elastic "constants" are absolutely frequency-dependent, whereas it is desirable to use only normalized frequency (1th) when there is a layer of thickness h contiguous to the substrate material.

A more convenient procedure is to implement a perturbation calculation in which the lossless (real) propagation velocity is first found, and the attenuation constant α_p is then computed from the associated strains and the viscosity tensor. This method has been described by King and Sheard, (2) and includes only the losses due to the Akheiser mechanism (3) (which dominates the temperature dependent losses in dielectrics).

The power dissipated per unit volume by the wave is given by twice the dissipation function:

$$2\psi = \sum_{i,j,k,\ell=1}^{3} \eta_{ijk\ell} \dot{S}_{ij} \dot{S}_{k\ell}^{*}$$
 (1)

where

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$
 (2)

are the tensor strain components and $\eta_{ijk\ell}$ are the viscosity tensor components. The viscosity tensor has the same symmetry as the elastic tensor. Note that the dissipation function depends on strain time derivatives rather than

particle velocity components $\mathring{u}_{\hat{i}};$ if the latter entered, a translation of the crystal would result in loss .

The attenuation constant α_p is the reciprocal of the distance (measured along the propagation vector, \vec{k}) over which the intensity of the wave decreases by a factor of e^{-1} . This is given by ⁽⁴⁾

$$\alpha_{p} = \frac{2 \int \psi \, dx_{3}}{P} \tag{3}$$

where P is the component of the time average acoustic flux along \vec{k} (the x_1 direction) and the x_3 axis is normal to the surface.

The computer programs of reference 1 compute P/α . The numerator, $2\int r \ dx_3$, is computed as follows. The particle displacement components of the surface wave, in one medium (substrate or layer) take the form

$$u_{i} = e^{j\omega t} \sum_{q} B^{(q)} \beta_{i}^{(q)} \exp \left\{ -j \frac{\omega}{v_{s}} \sum_{m=1}^{3} \gamma_{m}^{(q)} x_{m} \right\}$$
 (4)

In (4), $\gamma_1^{(q)} = 1$ and $\gamma_2^{(q)} = 0$ are direction cosines of propagation in the surface plane, and are independent of q. $\gamma_3^{(q)}$ is related to the q'th transverse wave number $\alpha^{(q)}$ by $\gamma_2^{(q)} = -j\alpha^{(q)}$. The use of this γ -notation simplifies the substitution of (4) and (' into (1), which gives the result

$$\frac{2}{\sqrt{3}} \int \psi \ dx_3 = \frac{1}{2v_s} \sum_{q,r} \sum_{i,j,k,\ell=1}^{3} \frac{\eta_{ijk\ell} B^{(q)}_{B^*(r)} \beta_i^{(q)} \beta_k^{*(r)} \gamma_j^{(q)} \gamma_\ell^{*(r)} Q_{qr}}{\alpha^{*(r)} + \alpha^{(q)}}$$
(5)

The B's, β 's, γ 's and α 's are all readily available in the computer programs of reference 1. The factor Q_{qr} is unity in a crystalline substrate but has the value $Q_{qr} = \exp\left\{\frac{\omega h}{v_s}\left[\sigma^*(r) + \alpha^{(q)}\right]\right\} - 1$ in a layer of normalized thickness ωh . q and r are summed over all the partial waves in each medium.

The new version of the program evaluates equation (5) as well as P/w and then calculates

$$\frac{\alpha_{p}}{\omega^{2}} = \frac{\left[\frac{2}{\omega^{3}}\int \psi \, dx_{3}\right]_{\text{substrate}} + \left[\frac{2}{\omega^{3}}\int \psi \, dx_{2}\right]_{\text{layer}}}{\left|\frac{2}{\omega}\right|_{\text{substrate}} + \left|\frac{P}{\omega}\right|_{\text{layer}}}.$$
 (6)

The terms for the layer are simply omitted when the layer is absent. Equation (6) displays explicitly the w^2 -dependence of the attenuation coefficient, in which α_p is given in nepers/meter. The program output contains 1.715 α_p/w^2 which gives α_p'/f^2 in dB cm⁻¹ Hz⁻². It also prints α_p'/f^2 (where $\alpha_p'' = 10^{-4} \alpha_p' v_s$, v_s in m/sec), thus expressing the attenuation in dB μ sec⁻¹ Hz⁻².

Results for Y-cut quartz are given in Figure 1. Values for the viscosity coefficients were taken from reference 5. α'/f^2 and $1/v_s = \beta/2\pi f$ are plotted versus direction of propagation in the XZ plane. The similar shapes of the curves indicate that the Q(= $\beta/2\alpha$) has relatively small anisotropy.

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APPENDIX - PROGRAMMING EXTENSIONS

The attenuation calculations have been added to the program⁽¹⁾ for piezoelectric crystals in the presence of an electric conducting plane and also to the cases of a perfectly conducting isotropic elastic film on the piezoelectric substrate. The corresponding program descriptions of reference 1 hold, subject to the following modifications.

I. Modifications to Input Data (see \$CONST data)

A. Changes to Existing Input Data

The elastic constants (C_{pq}) must be input in the following order:

$$G = C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{22}, C_{23}, C_{24}, C_{25}, C_{26},$$

$$C_{33}, C_{34}, C_{35}, C_{36}, C_{44}, C_{45}, C_{46}, C_{55}, C_{56}, C_{66}$$

These 21 input values, together with the 21 transformed elastic constants (C'_{ij}) are printed out in the order shown above whenever COEFF = .TRUE.

B. Additions to Input Data

In order to compute the attenuation coefficient the following parameters must be added to the input data:

$$\underline{\text{$CONST}}$$
 - Coefficients of Viscosity (r_{pq})

(Optional) Medium A (isotropic layer) – input two values of $\boldsymbol{\eta}$

ETAA =
$$\eta_{11}, \eta_{44}$$

which are used to generate a twenty-one (21) element array whose elements, in standard reduced-subscript notation are:

$$r_{15} = 0$$

$$\gamma_{34} = 0$$

$$r_{35} = 0$$

$$\eta_{36} = 0$$

$$\eta_{44} = \eta_{44}$$

$$\gamma_{45} = 0$$

$$\eta_{46} = 0$$

$$\gamma_{66} = \gamma_{44}$$
 where $\gamma_{12} = \gamma_{11} - 2\gamma_{44}$

Medium B (substrate) – input all 21 values of η in the order listed above

ETAB =
$$\eta_{11}, \eta_{12}, \eta_{13}, \dots, \eta_{22}, \eta_{23}, \dots, \eta_{66}$$

Wherever printout is requested, both the input constants and the transformed constants are tabulated in the order shown above.

\$INPUT

Input Name	Equation Name	Definition
FATEN		A logical parameter which controls the calculation of the Attenuation Coefficient
		TRUE Caiculate the Attenuation Coefficient (input 19's in the \$CONST data)
		.FALSE Do <u>not</u> calculate the Attenuation Coefficient.
		(Nominal Value = .FALSE)

II. Modifications to Output Data

A. Changes

- 1. The input elastic, piezcelectric and dielectric constants are included in the printout of the transformed constants.

 (The elastic constants have been re-ordered for both input and output see I.A.)
- 2. The <u>FINAL ANSWERS</u> are now normalized as follows: All field quantities (u, T/w, S/w, E/w, D/w) are first found, as before, with arbitrary normalization. From these amplitudes a value of (P_1/w) is found. The field quantities which are printed out are normalized through division by the factor

Medium B (substrate) – input all 21 values of η in the order listed above

$$ETAB = \eta_{11}, \eta_{12}, \eta_{13}, \dots, \eta_{22}, \eta_{23}, \dots, \eta_{66}$$

Wherever printout is requested, both the input constants and the transformed constants are tabulated in the order shown above.

\$INPUT

Input Name	Equation Name	Definition
FATEN		A logical parameter which controls the calculation of the <u>Attenuation Coefficient</u>
		TRUE Calculate the Attenuation Coefficient (input η's in the \$CONST data)
		.FALSE Do not calculate the Attenuation Coefficient.
		(Nominal Value = .FALSE.)

II. Modifications to Output Data

A. Changes

- 1. The input elastic, piezoelectric and dielectric constants are included in the printout of the transformed constants.
 (The elastic constants have been re-ordered for both input and output see I.A.)
- 2. The <u>FINAL ANSWERS</u> are now normalized as is lows: All field quantities (u, T/w, S/w, E/w, D/w) are first found, as before, with arbitrary normalization. From these amplitudes a value of (P_1/w) is found. The field quantities which are printed out are normalized through division by the factor

$$\sqrt{\operatorname{Re}\left(\frac{\mathbb{F}_{1}}{I}\right)} \quad \cdot \quad \left(\frac{\mathbb{E}_{1}(0)}{\operatorname{E}_{1}(0)}\right)$$

The fear time average power flow components, $(P_1/1)$ and $(P_2/1)$ for madia A and B, are each normalized to $Re(P_1/1)_B$. With the above normalization $(P_1/1)_B$ takes the value unity.

- B. Additions (included in FINAL ANSWERS only)
 - 1. In the substrate the normalized transverse wavenumbers $e^{(i)}$ and the corresponding amplitudes $e^{(i)}$ for each partial wave are printed.
 - 2. A tabular summary of the following quantities is printed for each value of propagation direction angle () when $\Delta v \neq 0$

$$v_s = 1/v_s = \alpha_p' = \alpha_p'' = \tan^{-1}x$$
 (degrees) (m/sec) (sec/m) (dB/cm Hz²) (dB/ μ sec Hz²)

tan x is the angle between the k vector and the mechanical power flow; thus

$$x = \begin{cases} \frac{Re \left(\frac{P_2}{i}\right)_A + Re \left(\frac{P_2}{i}\right)_B}{Re \left(\frac{P_1}{i}\right)_A + Re \left(\frac{P_1}{i}\right)_B} & \text{layer present} \\ \frac{Re \left(\frac{P_2}{i}\right)_B}{Re \left(\frac{P_1}{i}\right)_B} & \text{layer absent} \end{cases}$$

3. For the piezoelectric substrate with a perfectly conducting plane at a distance $\sin = 0$ and at a distance $\sin = \pi$, the following stored energy terms are printed: P_E , P_M , P_D , P_1 , F_2 . These terms are defined and explained in the following.

When the constitutive relations

$$S = S : T + d_t \cdot E$$

$$D = d \cdot T + \varepsilon \cdot E$$

are used, the total stored energy is given by the sum, $P_E + 2P_M + P_D$, where

$$P_{E} = \frac{1}{2} \int \overrightarrow{T} \cdot \overrightarrow{s} \cdot \overrightarrow{E} \cdot \overrightarrow{T} dv$$

$$2P_{M} = \frac{1}{2} \int \left\{ \overrightarrow{E} \cdot \overrightarrow{d} \cdot \overrightarrow{T} + \overrightarrow{T} : \overrightarrow{d} \cdot \overrightarrow{E} \right\} dv$$

$$P_{D} = \frac{1}{2} \int \overrightarrow{E} \cdot \overrightarrow{E} \cdot \overrightarrow{E} dv$$

The appropriate computational expressions are:

a) Elastic energy term evaluated using stresses

$$P_{E} = \frac{1}{2} \sum_{i,j=1}^{6} \sum_{k,\ell=1}^{4} f_{i}^{(\ell)} f_{j}^{*(k)} s_{ij} \frac{B^{(\ell)} B^{*(k)}}{\alpha^{(\ell)} + \alpha^{*(k)}}$$

b) Mutual energy term

$$2P_{M} = Re \sum_{i=1}^{6} \sum_{m=1}^{3} \sum_{k=1}^{4} \sum_{\ell=1}^{4} f_{i}^{(\ell)} g_{m}^{*(k)} d_{mi} \frac{g^{(\ell)} g_{k}^{*(k)}}{g_{m}^{(\ell)} + g_{m}^{*(k)}}$$

c) Electric energy term, dielectric constant at constant stress

$$P_{D} = \frac{1}{2} \sum_{m, n=1}^{3} \sum_{k, l=1}^{4} g_{m}^{(l)} g_{n}^{*(k)} \epsilon_{mn}^{T} \frac{g^{(l)} g_{n}^{*(k)}}{e^{(l)} + e^{*(k)}}$$

+
$$\begin{cases} 0, & \text{sh} = 0 \\ \frac{1}{4} \varepsilon_0 \mid \sum_{k=1}^{4} B^{(k)} \hat{s}_{4}^{(k)} \mid^2, & \text{sh} = \infty \end{cases}$$

where $f_i^{(\ell)}$, $g_m^{(k)}$, and $h_i^{(\ell)}$ are given below.

An alternative set of constitutive relations is

$$T = C^{*}: S - e \cdot E$$

When these are used, the total stored energy in the piezoelectric substrate decomposes into $P_1 + P_2$, where

$$P_I = \frac{1}{2} \int_{S}^{+\infty} C^E \cdot S dv$$

and

$$P_2 = \frac{1}{2} \int_{E}^{*} \stackrel{\longleftrightarrow}{\cdot} \stackrel{\circ}{\epsilon} \stackrel{\circ}{\cdot} \stackrel{\bullet}{E} dv$$

The appropriate computations expressions are:

d) Elastic energy term evaluated using strains

$$P_{1} = \frac{1}{2} \sum_{i,j=1}^{5} \sum_{k,\ell=1}^{4} h_{i}^{(\ell)} h_{j}^{*(k)} C_{ij} \frac{B^{(\ell)}B^{*(k)}}{\alpha^{(\ell)} + \alpha^{*(k)}}$$

e) Electric energy term, dielectric constant at constant strain

$$P_{2} = \frac{1}{2} \sum_{m, n=1}^{3} \sum_{k, \ell=1}^{4} g_{m}^{(\ell)} g_{n}^{*(k)} \varepsilon_{mn}^{S} \frac{B^{(\ell)} B^{*(k)}}{\alpha^{(\ell)} + \alpha^{*(k)}}$$

$$+ \begin{cases} 0, & \text{wh = 0} \\ \frac{1}{2} \varepsilon_{0} | \sum_{k=1}^{4} B^{(\ell)} \beta_{4}^{(\ell)} |^{2}, & \text{wh = } \infty \end{cases}$$

For the above five energy quantities we define:

C_{ij} = elastic constants (program input)

$$s_{ij} = (C^{-1})ij$$

$$d_{ik} = \sum_{j=1}^{6} e_{ij}^{s}_{jk}$$
 ($e_{ij} = program input$)

$$\epsilon_{1j}^{T} = \epsilon_{1j}^{S} + \sum_{k=1}^{6} d_{ik} e_{jk}$$
 ($\epsilon^{S} = \text{program input}$)

while the $f_i^{(\ell)}$, $g_m^{(k)}$, and $h_i^{(\ell)}$ are defined by:

$$f_{1}^{(\ell)} = \beta_{1}^{(\ell)} [-j C_{11} - \alpha^{(\ell)} C_{15}] + \beta_{2}^{(\ell)} [-j C_{16} - \alpha^{(\ell)} C_{14}]$$
$$+ \beta_{3}^{(\ell)} [-j C_{15} - \alpha^{(\ell)} C_{13}] + \beta_{4}^{(\ell)} [-j e_{11} - \alpha^{(\ell)} e_{31}]$$

$$\begin{split} f_2^{(\ell)} &= \beta_1^{(\ell)} [-jC_{12} - \alpha^{(\ell)}C_{25}] + \beta_2^{(\ell)} [-jC_{26} - \alpha^{(\ell)}C_{24}] \\ &+ \beta_3^{(\ell)} [-jC_{25} - \alpha^{(\ell)}C_{23}] + \beta_4^{(\ell)} [-je_{12} - \alpha^{(\ell)}e_{32}] \end{split}$$

$$\begin{split} f_{3}^{(\ell)} &= \bar{r}_{1}^{(\ell)} \left[-j C_{13} - e^{(\ell)} C_{35} \right] + \bar{r}_{2}^{(\ell)} \left[-j C_{36} - e^{(\ell)} C_{34} \right] \\ &+ \bar{r}_{3}^{(\ell)} \left[-j C_{35} - e^{(\ell)} C_{33} \right] + \bar{r}_{4}^{(\ell)} \left[-j e_{13} - e^{(\ell)} e_{33} \right] \end{split}$$

$$\begin{split} \mathbf{f}_{4}^{(\ell)} &= \mathbf{f}_{1}^{(\ell)} [-\mathbf{j} \mathbf{C}_{14} - \mathbf{e}^{(\ell)} \mathbf{C}_{45}] + \mathbf{f}_{2}^{(\ell)} [-\mathbf{j} \mathbf{C}_{46} - \mathbf{e}^{(\ell)} \mathbf{C}_{44}] \\ &+ \mathbf{f}_{3}^{(\ell)} [-\mathbf{j} \mathbf{C}_{45} - \mathbf{e}^{(\ell)} \mathbf{C}_{34}] + \mathbf{f}_{4}^{(\ell)} [-\mathbf{j} \mathbf{e}_{14} - \mathbf{e}^{(\ell)} \mathbf{e}_{34}] \end{split}$$

$$\begin{split} f_5^{(\ell)} &= \hat{s}_1^{(\ell)} [-j C_{15} - \alpha^{(\ell)} C_{55}] + \hat{s}_2^{(\ell)} [-j C_{56} - \beta^{(\ell)} C_{45}] \\ &+ \hat{s}_3^{(\ell)} [-j C_{55} - \alpha^{(\ell)} C_{35}] + \hat{s}_4^{(\ell)} [-j e_{15} - \alpha^{(\ell)} e_{35}] \end{split}$$

$$\begin{split} f_{6}^{(\ell)} &= \beta_{1}^{(\ell)} [-jC_{16} - \alpha^{(\ell)}C_{56}] + \beta_{2}^{(\ell)} [-jC_{66} - \alpha^{(\ell)}C_{46}] \\ &+ \beta_{3}^{(\ell)} [-jC_{56} - \alpha^{(\ell)}C_{36}] + \beta_{4}^{(\ell)} [-je_{16} - \alpha^{(\ell)}e_{36}] \end{split}$$

$$g_{\mathbf{I}}^{(k)} = \mathbf{j} \hat{\epsilon}_{4}^{(k)}$$

$$g_2^{(k)} = 0$$

$$g_3^{(k)} = \alpha^{(k)} \tilde{\gamma}_4^{(k)}$$

$$h_1^{(r)} = -_J \, \, F_1^{(\ell)}$$

$$\mathsf{h}_2^{(i)} = 0$$

$$h_3^{(\ell)} = -\alpha^{(\ell)}\beta_3^{(\ell)}$$

$$h_4^{(\ell)} = -\alpha^{(\ell)} \beta_2^{(\ell)}$$

$$h_5^{(\ell)} = [-\alpha^{(\ell)} \beta_1^{(\ell)} - j \beta_3^{(\ell)}]$$

$$h_6^{(\ell)} = -j \beta_2^{(\ell)}$$

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Additions to and modifications of a computer program for the numerical study of piezoelectric surface wave propagation are described. The main addition to the program is the calculation of surface wave attenuation using an expression based on a perturbation analysis and the viscosity tensor which accounts for losses due to the Akheiser mechanism. Other additions and modifications include the evaluation of stored energy expressions and changes in the format and normalization of the output data.

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